Using power spectra and Allan variances to characterise the noise of Zener-diode voltage standards

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Abstract: The differences between the voltage outputs of Zener-diode based electronic voltage standards (Zeners) and standard cells, between Zeners and a Josephson array voltage standard (JAVS) and between pairs of Zeners are measured. The data are analysed as time series. They are serially correlated so that the use of the usual expression for the standard deviation of the mean, the experimental standard deviation divided by the square root of the number of measurements, to characterise the dispersion is not justified. Noise in Zeners is of a $1/f$ nature and is characterised by using the power spectral density and Allan variance, a statistic used in time and frequency metrology. The uncertainty in the measurements of the 10V outputs of 13 Zeners is limited by $1/f$ noise to a 'floor' value ranging between two parts in 10^9 and eight parts in 10^9 of the nominal output value. Methods are proposed for estimating the Allan variance of a Zener using either a standard cell, a JAVS, or a small group of similar Zeners.

1 Introduction

This paper is one of a series (references to which are made in the following text) addressing the issue of serial correlations in electrical measurements. Correlations have an important influence on the characterisation of the scatter in a set of measurements. By repeating measurements under nearly identical circumstances, metrologists hope to reduce the uncertainty component due to random effects in the measurements. In particular, the dispersion of the mean value of the results of n repetitions of a measurement is traditionally characterised by the sample standard deviation s divided by $n^{1/2}$. This analysis is only justified if the observations are independent and have the same probability distribution. Alternatively, a histogram of the observations may have the appearance of a normal distribution and, when so, this may be incorrectly considered to imply that the standard deviation of the mean is given by $s\ n^{-1/2}$. In fact, even highly correlated data may have a histogram that appears to be normal. These general issues are treated in more detail elsewhere [1, 2].

This paper presents a specific application of some of the techniques described in [1] to the measurement of electronic voltage standards referenced to internal Zener diodes (Zeners). These instruments play a crucial role in DC voltage metrology at the very highest levels of accuracy. Indeed, nearly all DC voltage measurements are linked to Josephson standards through traceability chains containing Zeners. Zeners are measured directly with Josephson array voltage standards (JAVS) and are used as working standards to disseminate DC voltage standards. The most demanding use to which they are put is to serve as travelling standards between laboratories, particularly national metrology institutes, for the purpose of checking the agreement between different realisations of Josephson standards. The latter are capable of providing a 10V or 1.018V reference standard of voltage having combined standard uncertainties of 0.5nV or less. The main result presented in this paper is that $1/f$ noise in Zeners limits the uncertainty of all Zener voltage measurements to at least several parts in 10^9 of the output voltage. This means that laboratories that do not posses Josephson voltage standards and use Zeners as primary voltage standards cannot realistically expect to achieve voltage measurement uncertainties of one part in 10^8.

In previous work [3, 4] we used spectral analysis to identify $1/f$ noise in a few Zeners. Such noise imposes a practical limit on the accuracy of Zeners. The Zener voltage seems to retain some 'memory' of its previous values and to move away from them in a random way. In contrast to this behaviour, if each succeeding measurement value is independent of previous values, the noise is white. Although spectral analysis can identify the presence of $1/f$ noise, it does not directly yield a statistic comparable to the standard deviation of the mean. For over 30 years, the time and frequency community has been routinely using the Allan variance to describe scatter in correlated observations. (Many key papers are grouped in [5].) In this work we take up Allan's suggestion [6] to use the Allan variance to describe the scatter in measurements outside the domain of time and frequency.

We report here for the first time the results of a systematic study of a statistically significant group of Zeners (13 Fluke 732A and 732B instruments) and give values of the Allan deviations corresponding to the $1/f$ noise floor. The calculations of the Allan deviations are based on three methods. Allan deviations of 1.018V Zener output levels were calculated from data obtained by comparison with standard cells. The most accurate estimates of the Allan deviations of both 1.018V and 10V Zener levels were obtained from direct comparisons with the JAVS at the

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Bureau International des Poids et Mesures (BIPM). At the 1.018V level, comparisons with standard cells give Allan deviations that agree well with those obtained via the JAVS. Because most users of Zeners have access neither to a JAVS nor to good standard cells, we also studied a method using only pairwise measurements of groups of three Zeners. The results are in reasonable agreement with those obtained using a JAVS.

2 Data analysis

2.1 Power spectral density

We suppose that $y(t)$ is a continuous variable, in our case a voltage, measured at equally spaced time intervals. For samples taken over a frame time T_F, the power spectral density (PSD) $S_y(f)$ is given by

$$S_y(f) = \lim_{T_F \to \infty} \frac{2}{T_F} E\left[|Y(f, T_F)|^2\right] \qquad (1)$$

where $Y(f, T_F)$ is the finite Fourier transform of $y(t)$, f is the Fourier frequency, and the operator E denotes the expectation operator. When calculating the PSD, we believe that there are advantages to dividing up a long time interval into a small number n_F (between three and ten) of nonoverlapping adjacent intervals each of duration T_F and each containing N_0 measurements. Individual measurements are spaced over equal time intervals Δt. The precision of the PSD increases as $n_F^{-1/2}$ and averaging helps to distinguish real spectral features from noise. We estimate S_y using ([7] eqn. 11.102):

$$\hat{S}_y(f_k) = \frac{2}{n_F T_F} \sum_{i=1}^{n_F} |Y_i(f_k, T_F)|^2 \quad k = 0, 1, \ldots, N_0/2 \qquad (2)$$

The Fourier transform of $y(t)$

$$Y(f, T_F) = \int_0^{T_F} y(t) \exp(-2\pi i f t) dt$$

has units Vs. Consequently, the units of S_y are $V^2s = V^2/Hz$. Modern analysis software is capable of calculating fast Fourier transforms of time series of 100 000 points in about one minute using a PC. By using eqn. 2 to analyse data acquired by a DVM we effectively use the DVM as a low-frequency spectrum analyser.

2.2 Allan variance

The Allan variance provides a way of characterising measurement processes in the time domain. It is related to the PSD and it too can reveal correlations in time series. It is easier to calculate than the PSD and does not require calculation of Fourier transforms. Because the Allan variance has long been used in time and frequency metrology, its interpretation is well established. Commercial software is even available to calculate Allan variances [8].

Suppose that $y(t)$ is measured at equally spaced time intervals τ, that the mean value in the kth time interval is $\bar{y}_k$, and that there is no significant dead time between the end of the kth interval and the beginning of the $(k + 1)$th interval. For long-term Zener measurements, lasting a day or more, it may be desirable to first correct the raw data for linear drift. The Allan variance can be expressed as

$$\sigma_y^2(\tau) = \left\langle \left(\bar{y}_{k+1} - \bar{y}_k\right)^2 \right\rangle / 2 \qquad (3)$$

where the angular brackets denote an infinite time average. In practice, a finite number M of measurements are carried out with minimum intervals τ_0. Adjacent intervals with durations τ_0, $2\tau_0$, $3\tau_0$, ... can be constructed and the Allan variance can be estimated from ([9] p.75)

$$\hat{\sigma}_y^2(\tau) = \sum_{k=1}^{P_0} \left[\bar{y}_{k+1}(\tau) - \bar{y}_k(\tau)\right]^2 / 2P_0 \qquad (4)$$

where P_0 is the number of pairs of $\bar{y}$ given by Int$(M/n) - 1$, corresponding to $\tau = n\tau_0$. Eqn. 4 is the practical working formula for calculating the Allan variance. For each value of n, eqn. 4 is used to calculate $\hat{\sigma}_y^2(n\tau_0)$.

In time and frequency metrology it has been observed that random fluctuations may be modelled by the following power law:

$$S_y(f) = \sum_{\alpha=-2}^{2} h_\alpha f^\alpha \qquad (5)$$

where the PSD $S_y(f)$ is given at the Fourier frequency f, and the h_α are intensity coefficients. For Zener and voltmeter measurements we observe values of $\alpha = 0$ and $\alpha = -1$, corresponding to white noise and $1/f$ noise, respectively. The Allan variance is related to the PSD by the following integral expression:

$$\sigma_y^2(\tau) = 2 \int_0^{\infty} S_y(f) |H(f)|^2 \frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2} df \qquad (6)$$

where $H(f)$ is the unitless transfer function of the detector. In this work, one detector is a digital nanovoltmeter used without filters. The transfer function is unity up to a cutoff frequency f_c, and zero for higher frequencies, so that the upper integration limit in eqn. 6 is f_c. For the analogue nanovoltmeter used in this work, $|H(f)|^2$ is, to a good approximation, the transfer function of a low-pass filter $(1 + (f/f_0)^2)^{-1}$ with characteristic frequency f_0 and an equivalent noise bandwidth f_c of about 1 Hz. As is the practice in time and frequency metrology, our results will be presented as Allan (standard) deviations. The key to interpreting PSDs and Allan deviations [10] is given in Table 1. The tabulated expressions apply either to the case of a sharp

Table 1: Characteristics of stability regimes in voltage measurements

Name	PSD $S_y(f)$	Allan variance $\sigma_y^2(\tau)$	Allan deviation $\sigma_y(\tau)$
White noise	h_0	$h_0/2\tau$	$(h_0/2)^{1/2}\,\tau^{-1/2}$
		$\frac{2}{3}\pi^2 f_c^2 h_0 \tau$, $\omega_c\tau << 1^\dagger$	$(\frac{2}{3}\pi^2 f_c^2 h_0)^{1/2}\,\tau^{1/2\dagger}$
$1/f$ noise	$h_{-1}f^{-1}$	$2h_{-1}\ln 2$	$(2h_{-1}\ln 2)^{1/2}$
Random walk noise	$h_{-2}f^{-2}$	$\frac{2}{3}\pi^2 h_{-2}\tau$	$(\frac{2}{3}\pi^2 h_{-2})^{1/2}\,\tau^{1/2}$
Linear drift $y(t) = at$		$\frac{1}{2}a^2\tau^2$	$(a/\sqrt{2})\tau$
Sinusoidal perturbation $y(t) = A\cos(2\pi f_M t)$		$[A^2 \sin^4(\pi f_M \tau)]/(\pi f_M \tau)^2$	$[A \sin^2(\pi f_M \tau)]/(\pi f_M \tau) = A[1 - \cos(2\pi f_M \tau)]/(2\pi f_M \tau)$

† Short sampling times $\omega_c\tau << 1$ and single-pole filter with equivalent noise bandwidth f_c

cut-off frequency f_c or a single-pole filter having an equivalent noise bandwidth of f_c. The daggers note the case for white noise where the single-pole filter has a functional dependence different from that of the sharp cut-off filter. Table 1 includes two nonstochastic effects: linear drift and a parasitic oscillation of amplitude A and frequency f_M. The latter effect is observed in some of our data.

We always measure voltage differences between a Zener and a second standard. When the second standard is a standard cell or a JAVS, both of which have lower noise than the Zener [3, 4, 11], we may assume that all of the noise is from the Zener. However, if this approximation is not true, the quantity y in eqn. 4 is the average difference between two voltages, $U_p - U_q$, for, say, two Zeners p and q. What is more, if there are correlations between them [12], eqn. 4 should be modified to

$$\hat{\sigma}^2_{pq}(\tau) = \hat{\sigma}^2_p(\tau) + \hat{\sigma}^2_q(\tau) - \sum_{k=1}^{P_0}\left\{\left[\overline{y}_{p_{k+1}}(\tau) - \overline{y}_{p_k}(\tau)\right] \times \left[\overline{y}_{q_{k+1}}(\tau) - \overline{y}_{q_k}(\tau)\right]\right\} / P_0 \quad (7)$$

where the terms in the summation account for correlations between p and q. Suppose we add a third Zener r and measure the three possible pairwise voltage differences. If we assume that the correlations in eqn. 7 are negligible, we have

$$\hat{\sigma}^2_{pq}(\tau) = \hat{\sigma}^2_p(\tau) + \hat{\sigma}^2_q(\tau)$$

$$\text{and} \quad \hat{\sigma}^2_p(\tau) = \left(\hat{\sigma}^2_{pq}(\tau) + \hat{\sigma}^2_{pr}(\tau) - \hat{\sigma}^2_{qr}(\tau)\right)/2 \quad (8)$$

where the indices p, q and r are cyclic. The method of estimating the Allan variances of individual instruments by the pairwise comparison of three instruments is called the 'three-cornered hat' method. Eqn. 8 are only rough approximations and could lead to negative (unphysical) variances in some cases; they must be used with caution. In the present work we are able to check the validity of the assumption of negligible correlations by comparing results of Zener/Zener measurements with those of Zener/standard cell and Zener/JAVS measurements.

3 Measurement methods

The measurements were carried out on Fluke model 732A and 732B Zeners. Both provide output voltages of 1.018V and 10V. Within an instrument the two outputs are not independent; a resistive divider is used to derive the 1.018V signal from that at 10V. Both output voltages were studied in this work. Three measurement procedures are used to evaluate the noise of a Zener. The first is to connect the 1.018V output of the Zener in series opposition to a standard cell and to read the voltage difference using a Hewlett-Packard 34420A digital nanovoltmeter. A good standard cell has a characteristic noise similar to the thermal noise of a resistor of a few hundred ohms. A typical run may consist of 8192 measurements made at a rate of 0.13s per reading. The second and most accurate procedure is to compare the Zener output voltages with those of a BIPM JAVS using an EM N1a analogue nanovoltmeter (on the 10 or 3µV ranges) and record the readings using a digital voltmeter connected to the output via an optically coupled isolation amplifier. A typical run consists of four frames of 8192 points each, taken at a rate of one point every 0.52 or 0.12s, depending on the time required for the DVM to acquire a single point. Thus, in some cases the JAVS must deliver a continuously stable voltage for over one hour. In the third procedure two Zeners are compared at 1.018V or at 10V. The measurement process is almost identical to that used with the standard cell, but the data are analysed using the three-cornered hat method.

In all three procedures, each point consists of a voltage and the corresponding time. Data are stored on a PC controlling the measurements and several runs can be strung together to form a longer measurement series. Any such measurements are sensitive to perturbations due to temperature variations (which may be cyclic) and to electromagnetic interference (EMI). Such effects were frequently monitored by using the nanovoltmeters to measure the signals obtained with the voltage standards replaced either by a wirewound resistor or by a short circuit.

4 Results

Fig. 1 shows the Allan deviation as a function of sampling time for measurements of the voltage difference between a 1.018V Zener output and a standard cell using an H-P 34420A. For sampling times below 10s, the Allan deviation decreases nearly as $\tau^{-1/2}$, indicating that in this range the noise is almost white. (In fact, the PSD shows a perturbation at 1.6Hz which appears as an increase of the Allan deviation near 1/1.6Hz = 0.6s.) The Allan deviation becomes nearly flat over the range from 20 to 320s, indicated by the heavy horizontal line in Fig. 1. This is the $1/f$ noise floor and its experimental value is estimated by averaging the Allan variances over the corresponding range of sampling times, giving a value of 0.0089µV. This is indicated by the horizontal dashed line. Another quasiperiodic perturbation, this one having a period between 500 and 1000s, is probably due to cycling of the laboratory air conditioning. The observation of the noise floor after only 20s of sampling time is somewhat surprising. It means that for this nanovoltmeter no increase in accuracy is gained by extending the Zener measurements over times greater than 20s.

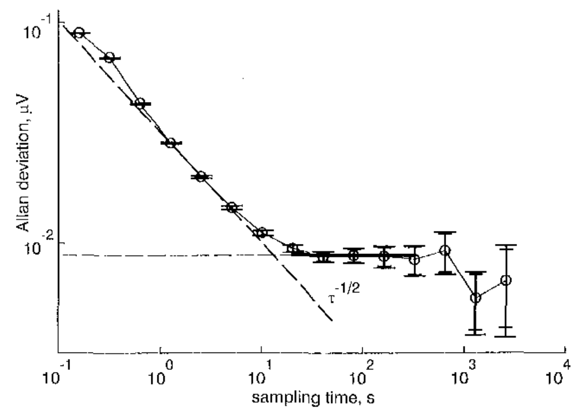

Fig. 1 *Allan deviation against sampling time for measurement of Zener B4 with respect to a standard cell*
Bars represent 1-σ confidence intervals; longer caps correspond to white noise, shorter caps to $1/f$ noise. The Allan deviation of the $1/f$ noise floor, which appears as a horizontal line in an Allan deviation plot, is estimated from Allan variances of the five circles crossed by the thick black line. The dashed diagonal line indicates the expected slope for pure white noise

The most accurate method we found to evaluate the $1/f$ noise of the Zeners is to directly compare them to a JAVS at 1.018 or 10V. Since the output voltage of the JAVS can be adjusted to nearly equal that of the Zener, a sensitive analogue detector, the EM N1a operating on the 1, 3 or 10µV ranges, was used for these measurements. Fig. 2*a* shows the Allan deviation calculated from the measurement of the 10V output of a Zener with the JAVS and the N1a. This detector is fitted with an input filter that imposes a time constant in its response to input voltages so that at short sampling times its response to white noise is approximately that of a single-pole filter. This is indicated in Table 1 with a dagger. For the shortest sampling times, the Allan deviation approaches a $\tau^{1/2}$ dependence. Beginning at a sampling time of 2s, the $1/f$ noise floor of the Zener is already attained. In this case, by the usual method of averaging the Allan variances in the $1/f$ noise regime, we find a $1/f$ noise floor of the Allan deviation equal to 0.068μV. Comparison with Fig. 1 illustrates that the $1/f$ noise floor is a characteristic of the Zener and the sampling time required to achieve the noise floor depends on the PSD of the detector.

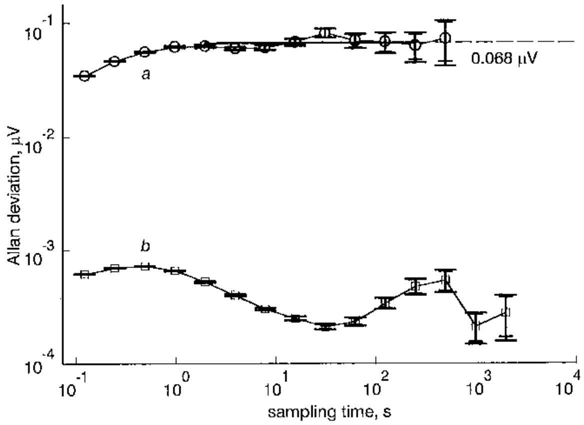

Fig. 2 *Allan deviation for comparison of Zener B8 at 10V with JAVS and for analogue nanovoltmeter used with input short-circuited*
Horizontal line represents $1/f$ noise floor
a Zener
b Short-circuited detector

Fig. 2*b* shows the Allan deviation for the N1a detector with the input terminals shorted. It shows the effect of the input filter at small sampling times. This is followed by a white noise regime in the time range from 1 to 10s. At longer times we see the effects of a periodic perturbation due to cycling of the air conditioning with a period of 1000s (16 min). Throughout the sampling time range, the Allan deviation of the detector remains well below that of the Zener. In the 10V measurements the equivalent noise resistance of the analogue nanovoltmeter was 35Ω and is much greater than the source resistance of the 10V outputs of the Zeners or the JAVS.

Fig. 3*a* shows the first 1024 measured voltage differences between the 10V Zener and the JAVS. This is a portion of the raw data from which the Allan deviation given in Fig. 2*a* is calculated. Figs. 3*b*, *c* and *d* show the smoothing effect obtained by averaging adjacent groups of 4, 4^2 and 4^3 points. The persistent skeleton due to the $1/f$ noise can clearly be seen.

Fig. 4 shows the PSD for the case of the short-circuited analogue nanovoltmeter. It is calculated from the time series that gives the Allan deviation shown in Fig. 2*b*. The high-frequency roll-off above 0.5Hz is due to the input filter. The PSD is flat in the white noise regime. This is followed by a poorly defined peak appearing slightly below 0.001Hz. For any time series the spectrum calculated for the widest possible frequency span, as in Fig. 4*a*, has a ragged appearance. Increasing the measuring time only increases the frequency span. Dividing the original time series into n_F frames and averaging spectra as in eqn. 2 yields smoothed spectra revealing high-frequency features nearly lost in the widest-span spectrum. The cost is the increase of the low-frequency limit by a factor of $n_F/2$. Curves *b* and *c* of Fig. 4 show the effects of averaging 15 frames each of 2^{12} points and 255 frames each of 2^8 points, respectively. They show more clearly the roll-off, a perturbation of fundamental frequency 0.0827Hz and its odd harmonics up to the 13th. For clarity, the spectra in Figs. 4*b* and *c* are displaced vertically by dividing by factors of 10 and 100, respectively.

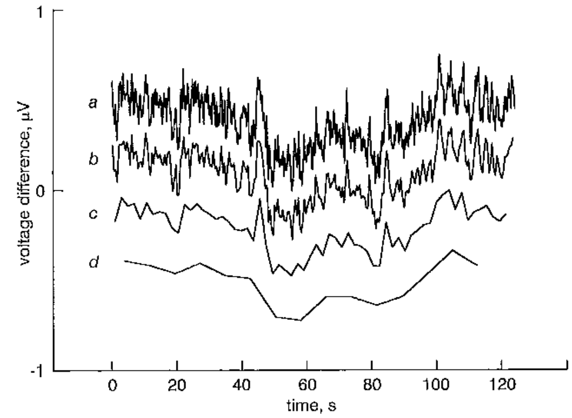

Fig. 3 *Chronological plot of first 1024 measured voltage differences between 10V Zener and JAVS from which is calculated the Allan deviation in Fig. 2a*
b, *c* and *d* show smoothing effect obtained by averaging successive groups of 4, 4^2 and 4^3 points, respectively. The persistent skeleton due to the $1/f$ noise can clearly be seen. Successive plots are displaced vertically for clarity

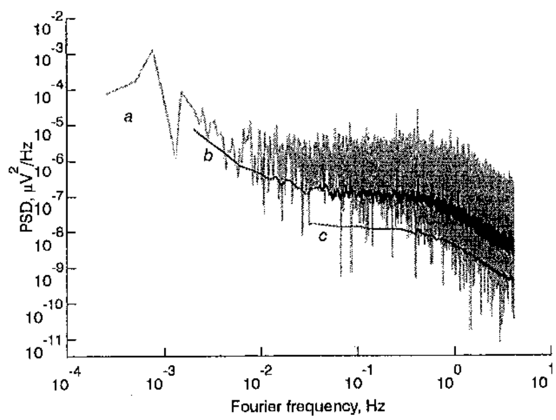

Fig. 4 *PSD corresponding to case of short-circuited analogue nanovoltmeter from the data that led to Fig. 2b*
a One frame of 2^{15} points
b 15 frames each of 2^{12} points
c 255 frames each of 2^8 points
Curves *b* and *c* are vertically displaced by factors of 10 and 100, respectively

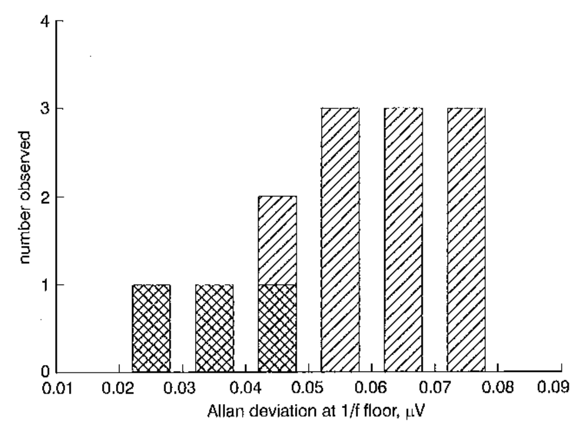

Fig. 5 *Summary of Allan deviations at 1/f floor for 10V outputs of 13 732B instruments deduced from JAVS measurements*
Cross-hatched shading corresponds to values from instruments fitted with M-type Zener references

Fig. 5 summarises the results of our determinations from JAVS comparison data of the Allan deviation of the $1/f$ noise floor for the 10V outputs of 13 732A and 732B Zeners. The values lie between 0.02 and 0.08μV. Previous information supplied by the manufacturer allows us to identify the basic reference/amplifier elements as M-type or L-type according to the component supplier [13]. We observe that the three instruments fitted with the L-type reference/amplifier (indicated by the cross-hatching) have a lower $1/f$ noise floor.

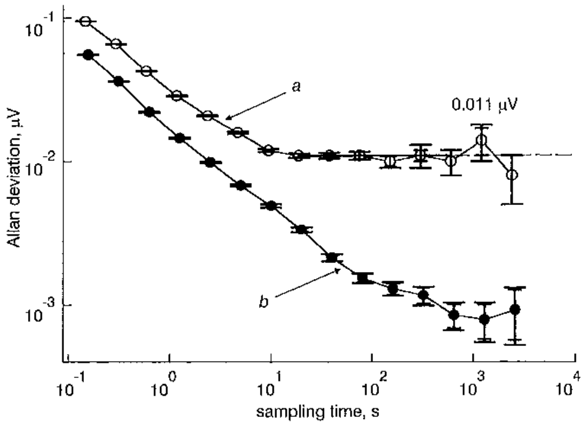

Fig. 6 *Allan deviation for measured voltage difference between 1.018V outputs of Zeners B4 and B9 for which the total source resistance is 2kΩ and for voltages measured across 2kΩ resistor with same DVM*
Horizontal line represents $1/f$ noise floor for the two Zeners. This is one element of a three-cornered hat
a Zeners
b 2kΩ resistor

Fig. 6*a* represents the estimated Allan deviation for the measurement in which two 1.018V Zener outputs are compared using the DVM as the detector. The measurement is an element of the three-cornered hat method. Again, at small sampling times a slight increase in the noise above the white noise level can be perceived. The Allan deviation of the $1/f$ noise floor is estimated from the mean Allan variances over the range indicated by the heavy horizontal line. The dashed horizontal line indicates a $1/f$ noise floor value of 0.011μV. Fig. 6*b* shows the Allan deviation obtained from measurements using the same DVM with a 2kΩ resistor on its input so that the source resistance is the same as that of the Zener measurement in Fig. 6*a*. Although the Allan variance of the detector appears to tend toward a constant value for long sampling times, this limiting value is an order of magnitude below the $1/f$ noise floor of the Zeners so we rule out the possibility of the $1/f$ noise being due to the detector. The enhanced white noise in Fig. 6*a* over that of Fig. 6*b* is believed to be due mainly to current noise in the Zeners.

The PSD for the Zener measurement of Fig. 6*a* is shown in Fig. 7. It illustrates the advantages of using both the PSD and the Allan variance to characterise noise and to determine the time span of the 'flat' part of the Allan deviation. A least-squares fit to the low-frequency part is shown as a straight line in Fig. 7; the calculated slope is −0.93, a value close enough to −1 to quality this as $1/f$ noise. A 'corner frequency' separating the white noise regime from the $1/f$ regime occurs near 0.065Hz. In the Allan deviation plot the reciprocal of this frequency, 15s, corresponds to the approximate end of the $\tau^{-1/2}$ dependence and the onset of the τ^0 dependence seen in Fig. 6*a*. A perturbation of frequency 2.4Hz appears as a peak in the PSD.

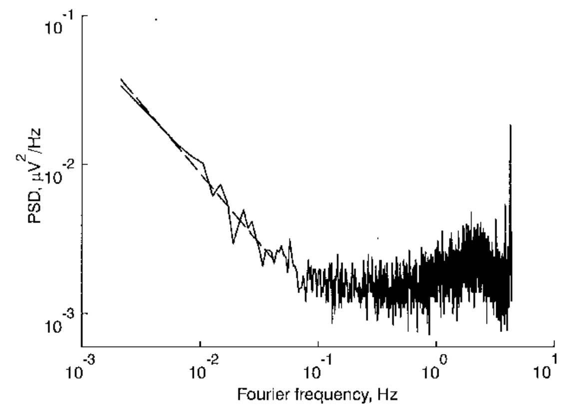

Fig. 7 *PSD for Zener/Zener comparison data that led to Allan deviation of Fig. 6a resulting from mean of 19 frames each of 2^{12} points*
Broken line through the first 20 points results from a least-squares fit that gives a slope of −0.93. In such a logarithmic plot the PSD for $1/f$ noise has a slope of −1. A periodic perturbation appears at 2.4Hz

Fig. 8*a* shows a portion of the raw data, measured voltage difference against measuring time from which the functions shown in Figs. 6 and 7 were calculated. In Figs. 8*b* to *e* these data are averaged into groups of 4, 4^2, 4^3 and 4^4 successive points, respectively. If the noise were white, in moving from *a* to *e* the dispersion in each plot would decrease by a factor of two with respect to the plot above it. This seems to be true in moving from *a* to *c* but this is not so when moving from *c* to *e*. The persistence of an irregular 'skeleton' is a characteristic of $1/f$ noise. The data in *a* were corrected for a drift rate of about 1pV/s so that the correction at the end of the time span shown in Fig. 8 was 1.4nV and could have been ignored in this case. Such a drift would be nearly imperceptible in Fig. 8.

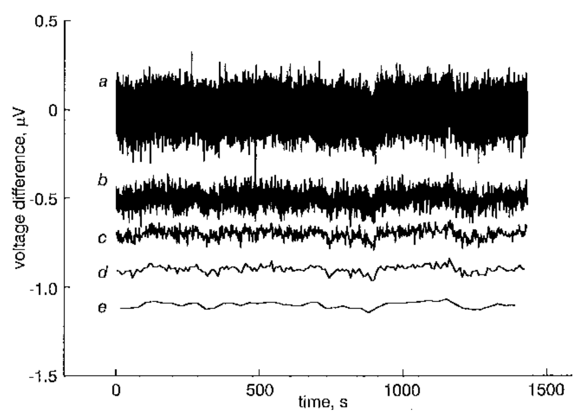

Fig. 8 *First 12288 measured voltage differences between two Zeners at 1.018V*
This is a portion of the data from which Figs. 6 and 7 were calculated. In *b*–*e* data are averaged into groups of 4, 4^2, 4^3 and 4^4 successive points, respectively; successive plots are shifted downward for clarity

Table 2 compares the results of our determinations of $\hat{\sigma}_{fl}$ for 1.018V outputs of three Zeners (under battery power) via comparisons with good standard cells with those obtained from comparisons with the JAVS, and shows that the agreement is quite good.

Table 3 compares the results of our determinations of $\hat{\sigma}_{fl}$ for 1.018V outputs of three Zeners, fitted with L-type reference/amplifiers, via the three-cornered hat method with the more accurate estimates obtained from comparisons with the JAVS. For these measurements the instruments were operated under internal battery power. The agreement is quite good. Table 4 compares the results of our determinations of the Allan deviation at the $1/f$ noise floor $\hat{\sigma}_{fl}$

obtained for 10V outputs of three Zeners (operated under internal battery power) via the three-cornered hat method with the more accurate estimates obtained from comparisons with the JAVS. Again the agreement is good. The results in Tables 3 and 4 imply that our assumption of negligible Allan covariance between pairs of Zeners may be a good approximation.

Table 2: Comparison of determinations of Allan deviations at 1/f noise floor $\hat{\sigma}_{fl}$ of 1.018V Zeners obtained by comparison with good standard cells (SC) with those obtained by comparison with Josephson standard (JAVS)

Zener	S, μV	J, μV	$100 \times (S-J)/J$
B4	0.0089	0.0065	37
B8	0.0071	0.0074	−4
B9	0.0073	0.0073	0

$S = \hat{\sigma}_{fl}$(SC), $J = \hat{\sigma}_{fl}$(JAVS)

Table 3: Comparison of determinations of Allan deviations at 1/f noise floor $\hat{\sigma}_{fl}$ of 1.018V Zeners obtained by three-cornered hat method (3CH) with those obtained by comparisons with Josephson standard (JAVS)

Zener	C, μV	J, μV	$100 \times (C-J)/J$
B4	0.0074	0.0065	14
B8	0.0097	0.0074	31
B9	0.0079	0.0073	8

$C = \hat{\sigma}_{fl}$(3CH), $J = \hat{\sigma}_{fl}$(JAVS)

Table 4: Comparison of determinations of Allan deviations at 1/f noise floor $\hat{\sigma}_{fl}$ of 10V Zeners obtained by three-cornered bat method (3CH) with those obtained by comparisons with Josephson standard (JAVS)

Zener	C, μV	J, μV	$100 \times (C-J)/J$
B1	0.055	0.044	25
B7	0.072	0.066	9
B8	0.076	0.071	7

$C = \hat{\sigma}_{fl}$(3CH), $J = \hat{\sigma}_{fl}$(JAVS)

This paper proposes methods for examining and quantifying $1/f$ noise in Zeners but does not propose a mechanism. An old report of $1/f$ noise in a component Zener diode of a type different from those used in the 732 instruments is given in the literature [14].

5 Conclusions

For the Zeners we tested the random measurement uncertainty is ultimately limited by $1/f$ noise, successive measurements are correlated, and prolonging measurement times cannot reduce the uncertainty component due to random effects in the measurements. Using currently available nanovoltmeters, the $1/f$ noise floor is attained in times of tens of seconds or less. The Allan variance provides a way of expressing measurement scatter in cases where serial correlations are important. Relative to the output voltages, the noise floor corresponds to about 7 parts in 10^9 for the 1.018V outputs of three (type-L) Zeners and from 2 parts in 10^9 to 8 parts in 10^9 for the 10V outputs of 13 Zeners. We show that the three-cornered hat method can be used to roughly estimate the Allan deviation of individual Zeners from pairwise Zener measurements, even under the simplifying assumption that Allan covariances are negligible. This offers the possibility of determining Allan deviations without recourse to either a Josephson standard or a stable standard cell. Finally, we find evidence that $1/f$ noise in Zeners varies with manufacturer of the component Zener diode. This may mean that some improvement in the manufacturing process may be possible to lower the accuracy limit on Zeners imposed by $1/f$ noise.

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