Measurement uncertainty in the $1/f$ noise region: Zener voltage standards

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Abstract

The measurement uncertainty is studied in terms of the Allan variance when the uncertainty is limited by $1/f$ noise. The deadtime in measurements determines the time scale of best measurement uncertainty. About ten equally spaced samples seem to map the noise spectrum well enough and no uncertainty improvement results from further averaging. The voltage difference of two Zener standards in the frequency band 10^{-6} Hz – 10^3 Hz was studied and closely $1/f$ behaviour was detected. The noise floor of one Zener is about 70 nV.

Introduction

Theoretically, the noise in many measurements can be described as $1/f$ noise limited by white noise at high frequencies. It is often understood that the $1/f$ corner frequency f_0 corresponds to an averaging time after which the measurement result does not improve. On the other hand, in metrology it is common to average hundreds of single measurements independent of the noise properties of the device under test. Also, often the standard deviation of the mean is used as a measure of uncertainty without detailed knowledge of the noise properties.

Properties of signal fluctuations are commonly described by their power spectral density (PSD). In frequency metrology, another quantity – the Allan variance is often used[1]. In a way, the Allan variance estimates the future uncertainty of result of the measurement based on its present value.

The long term (> 1 month) behaviour of Zener voltage standards is studied in this work both in terms of the PSD and the Allan variance. The noise PSD of Zener references has been studies earlier in e.g. Refs. [2] and [3].

Theory

We assume that the device is described by the spectral density function $S(f) = S_0(1 + f_0/f)$. In practice, the results are corrupted by digital sampling and filtering. Severe aliasing and leakage may be introduces especially when a DVM is the measuring instrument.

Let us consider a measurement with deadtime shown in the insert of Fig. 1, where τ is the integration time

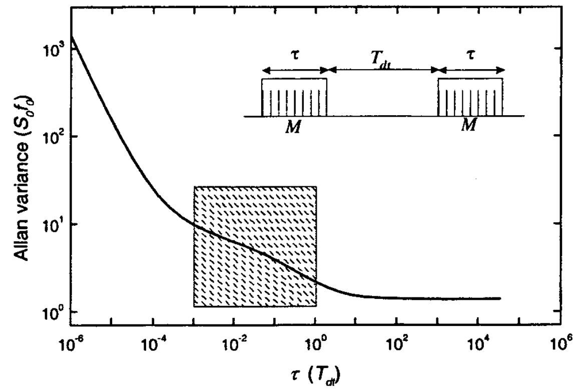

Fig. 1. Allan variance with deadtime. $T_{dt} = 1000/(4\ln 2 \cdot f_0)$. Insert: measurement procedure. Region 2 is shadowed (see text).

and $\tau + T_{dt}$ is the total duration of one measurement. The corresponding Allan variance can be calculated from $S(f)$: [4]

$$\sigma_y^2(2,\tau;T_{dt}) = \frac{2}{\pi\tau}\int_0^\infty S(\frac{2u}{\tau})\frac{\sin^2 u}{u^2}\sin\left[(1+\frac{T_{dt}}{\tau})u\right]du. \quad (1)$$

Typically this could correspond to a measurement with a DVM ($T_{dt} \sim 1$ s) or to an international comparison ($T_{dt} \sim 1 - 7$ d).

A curve calculated from Eq. 1 is shown in Fig. 1. The behaviour of σ_y^2 is divided into three regions:

1. white noise region $\tau < (4\ln 2 \cdot f_0)^{-1}$, where the variance diverges as $1/\tau$,
2. deadtime limited region $(4\ln 2 \cdot f_0)^{-1} < \tau < T_{dt}$, where logarithmic behaviour is seen, and
3. $1/f$ noise region $\tau > T_{dt}$, where the Allan variance becomes constant.

In other words: to minimize the measurement uncertainty, one should average at least as long as the deadtime in the measurement, if $T_{dt} > (4\ln 2 \cdot f_0)^{-1}$. However, after $\tau \approx 3 \cdot T_{dt}$, not much can be gained by further averaging.

Measurements and results

To test the $1/f$ noise hypothesis, the voltage difference between the 10 V outputs of two Fluke 732B Zener voltage references was measured with a Keithley 182

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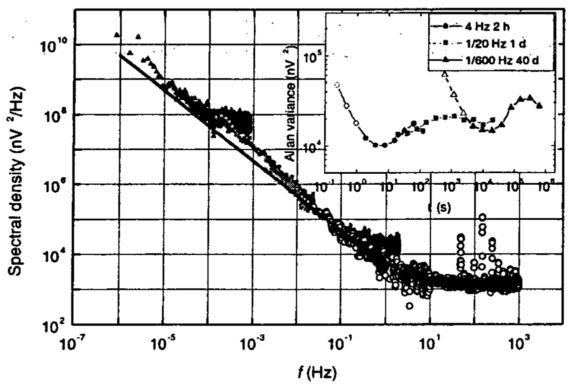

Fig. 2. Spectral density and Allan variance of 10-V Zener voltage fluctuations.

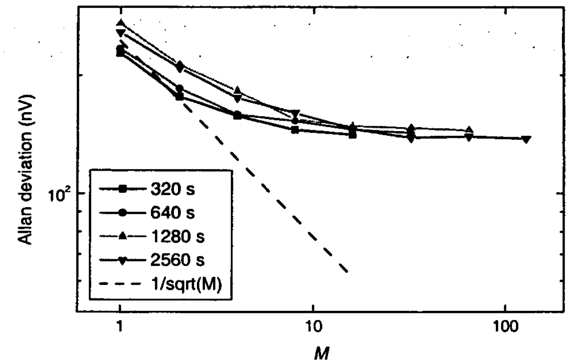

Fig. 3. Allan deviation as a function of number of averages within different time separations τ (measurement 1/20 Hz, 1 d). Dashed line shows the $1/\sqrt{M}$ behaviour of white noise.

nanovoltmeter. No filtering was used except the setting NPLC 1. Three sets of measurements were obtained, in the first the sampling rate was 4 Hz and duration 2 h, in the second 1/20 Hz and 1 day and in the third 1/600 Hz and 40 days, correspondingly. Linear drift was subtracted from the data before the analysis. In addition, the 'high' frequency spectrum was measured with a spectrum analyzer and a low noise preamplifier.

The power spectral density and Allan variance of Zener voltage fluctuations are shown in Fig. 2. Corner frequency $f_0 = 3.5$ Hz and spectral density $S_0^{1/2} = 36.7$ nV/$\sqrt{\text{Hz}}$ were obtained from the spectrum analyser data (open circles). Only a small deviation from this $1/f$ behaviour (solid curve) is observed even at the smallest frequencies and it can be attributed to environmental variations and sampling effects (compare the insert of Fig 2).

In the insert of Fig. 2 the Allan variance of the data is plotted. The three measurements with different sampling times fit again well together excluding the first points of the 40 d data. This is due to 'aliasing' of $1/f$ noise because of the large deadtime present. There seems to be a shallow minimum at 1-10 s intervals, but again the first points of the 4 Hz curve are dominated by deadtime. At the largest time intervals, the variance slightly increases, which may indicate variations in the laboratory conditions at $\geq$ 1-d time scales.

In Fig. 3, the variance of the data of the second measurement set is shown, when M equally spaced data points within interval τ are averaged and the variance of such subsequent averages is plotted as a function of M for different values of τ. Now, in principle $T_{dt} \simeq 0$, but there is substantial, increasing deadtime between individual nonaveraged data points as τ increases (see the insert in Fig. 1). The curve shapes are almost identical: first the variance decreases but the limiting Allan variance is obtained when $M \simeq 10$. Evidently, the variance of the mean, which assumes a $1/M$ decrease, would lead to a considerably underestimated type A uncertainty.

Conclusions

Development of present day instrumentation enables measurements down to the fundamental noise limit of metrological devices. In the $1/f$ noise region, new types of measurement practices are needed. It appears useless to average more than about 10 samples. In, e.g., international comparisons, the deadtime spent on transportation and instrument stabilization should be minimized and the measurement time need not be more than three times this deadtime. Repeated fast comparisons yield smaller uncertainty than one 'slow' comparison.

Compared to Ref. [3], we observe very closely $1/f$ behaviour at low frequencies. Measurement of a Zener voltage standard seems to be almost invariably dominated by its $1/f$ noise. With a typical 10-V Zener standard, the attainable accuracy in, e.g. voltage comparisons is at best 70 nV. We intend to continue the study to even longer measurement periods to see the effect of environmental variations on the observed noise.

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